# Notice of Allowability	Application No. 09/654,214	Applicant(s) KLIEGEL, JAMES R.	
	Examiner Herng-der Day	Art Unit 2128	

-- The MAILING DATE of this communication appears on the cover sheet with the correspondence address--

All claims being allowable, PROSECUTION ON THE MERITS IS (OR REMAINS) CLOSED in this application. If not included herewith (or previously mailed), a Notice of Allowance (PTOL-85) or other appropriate communication will be mailed in due course. **THIS NOTICE OF ALLOWABILITY IS NOT A GRANT OF PATENT RIGHTS.** This application is subject to withdrawal from issue at the initiative of the Office or upon petition by the applicant. See 37 CFR 1.313 and MPEP 1308.

1. ☒ This communication is responsive to _Amendment received 10/5/05_.

2. ☒ The allowed claim(s) is/are _19-20, now renumbered as 1-2_.

3. ☐ Acknowledgment is made of a claim for foreign priority under 35 U.S.C. § 119(a)-(d) or (f).
   a) ☐ All b) ☐ Some* c) ☐ None of the:
   1. ☐ Certified copies of the priority documents have been received.
   2. ☐ Certified copies of the priority documents have been received in Application No. _____ .
   3. ☐ Copies of the certified copies of the priority documents have been received in this national stage application from the International Bureau (PCT Rule 17.2(a)).

   * Certified copies not received: _____.

Applicant has THREE MONTHS FROM THE "MAILING DATE" of this communication to file a reply complying with the requirements noted below. Failure to timely comply will result in ABANDONMENT of this application.
THIS THREE-MONTH PERIOD IS NOT EXTENDABLE.

4. ☐ A SUBSTITUTE OATH OR DECLARATION must be submitted. Note the attached EXAMINER'S AMENDMENT or NOTICE OF INFORMAL PATENT APPLICATION (PTO-152) which gives reason(s) why the oath or declaration is deficient.

5. ☐ CORRECTED DRAWINGS (as "replacement sheets") must be submitted.
   (a) ☐ including changes required by the Notice of Draftsperson's Patent Drawing Review (PTO-948) attached
   1) ☐ hereto or 2) ☐ to Paper No./Mail Date _____.
   (b) ☐ including changes required by the attached Examiner's Amendment / Comment or in the Office action of Paper No./Mail Date _____.

Identifying indicia such as the application number (see 37 CFR 1.84(c)) should be written on the drawings in the front (not the back) of each sheet. Replacement sheet(s) should be labeled as such in the header according to 37 CFR 1.121(d).

6. ☐ DEPOSIT OF and/or INFORMATION about the deposit of BIOLOGICAL MATERIAL must be submitted. Note the attached Examiner's comment regarding REQUIREMENT FOR THE DEPOSIT OF BIOLOGICAL MATERIAL.

Attachment(s)

1. ☐ Notice of References Cited (PTO-892)
2. ☐ Notice of Draftperson's Patent Drawing Review (PTO-948)
3. ☐ Information Disclosure Statements (PTO-1449 or PTO/SB/08), Paper No./Mail Date _____
4. ☐ Examiner's Comment Regarding Requirement for Deposit of Biological Material
5. ☐ Notice of Informal Patent Application (PTO-152)
6. ☒ Interview Summary (PTO-413), Paper No./Mail Date _20060622_ .
7. ☒ Examiner's Amendment/Comment
8. ☒ Examiner's Statement of Reasons for Allowance
9. ☐ Other _____.

KAMINI SHAH
SUPERVISORY PATENT EXAMINER

DETAILED ACTION

1. This communication is in response to Applicant's Amendment to Office Action dated December 21, 2004, mailed September 30, 2005, and received by PTO October 5, 2005.

1-1. Claims 19 and 20 have been examined and allowed.

EXAMINER'S AMENDMENT

2. An Examiner's amendment to the record appears below. Should the changes and/or additions be unacceptable to applicant, an amendment may be filed as provided by 37 CFR 1.312. To ensure consideration of such an amendment, it MUST be submitted no later than the payment of the issue fee.

3. Authorization for this Examiner's amendment was given in a telephone interview with Mr. George W Hoover (Reg. No. 32,992) on June 21, 2006.

4. The specification has been amended as follows:

4-1. At page 16, line 6, replaces "$\left(\overline{c_1^{l+1} c_2^m c_3^n} - \overline{c_1^{l+1} c_2^m c_3^n}, \text{etc.} \right)$" as "$\left(\overline{c_1^{l+1} c_2^m c_3^n}^{*} - \overline{c_1^{l+1} c_2^m c_3^n}, \text{etc.} \right)$".

5. The application has been amended as follows:

5-1. Replace claim 19 as follows:

19. (Currently Amended) A method of analyzing and computing ~~modelling~~ anisotropic turbulent flow quantities ~~in~~ of an anisotropic fluid comprising:

providing input to a general purpose computer defining, for an anisotropic fluid~~,~~ a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

instructing the general purpose computer to calculate n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations ~~closure relationships~~ using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

using the set of closed time average turbulent moment equations to ~~predict~~ calculate a ~~anisotropic~~ turbulent flow quantity of the anisotropic fluid; and

displaying the calculated turbulent flow quantity;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional ~~Turbulent~~ Kinetic Energy

$$\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right]+\overline{u}_1\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\overline{u}_2\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right]+\overline{u}_3\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]$$
$$+\ 2\left[\overline{u_1'^2}\frac{\partial\overline{u}_1}{\partial x_1}+\overline{u_1'u_2'}\frac{\partial\overline{u}_1}{\partial x_2}+\overline{u_1'u_3'}\frac{\partial\overline{u}_1}{\partial x_3}\right]$$
$$+\ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{\rho u_2'u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{\rho u_3'u_1'^2}\right]\right]$$
$$=\ 0$$

~~Turbulent~~ Shear

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right] + \overline{u}_1 \frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u}_2 \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u}_3 \frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right] \\
&+ \overline{u_1'u_2'}\frac{\partial \overline{u}_1}{\partial x_1} + \overline{u_2'^2}\frac{\partial \overline{u}_1}{\partial x_2} + \overline{u_2'u_3'}\frac{\partial \overline{u}_1}{\partial x_3} \\
&+ \overline{u_1'^2}\frac{\partial \overline{u}_2}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u}_2}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u}_2}{\partial x_3} \\
&+ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_1'u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_1'u_2'u_3'}\right]\right] \\
&= 0
\end{aligned}
$$

Directional ~~Turbulent~~ Kinetic Energy Fluxes

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'u_1'^2}\right] + \overline{u}_1 \frac{\partial}{\partial x_1}\left[\overline{u_1'u_1'^2}\right] + \overline{u}_2 \frac{\partial}{\partial x_2}\left[\overline{u_1'u_1'^2}\right] + \overline{u}_3 \frac{\partial}{\partial x_3}\left[\overline{u_1'u_1'^2}\right] \\
&+ 3\left[\overline{u_1'u_1'^2}\frac{\partial \overline{u}_1}{\partial x_1} + \overline{u_2'u_1'^2}\frac{\partial \overline{u}_1}{\partial x_2} + \overline{u_3'u_1'^2}\frac{\partial \overline{u}_1}{\partial x_3}\right] \\
&- 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right] \\
&+ 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right] \\
&+ 3\left[\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right] \\
&+ \frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_1'^2}\right] \\
&+ \left[\overline{u_1'^2u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\rho}\frac{\partial \overline{\rho}}{\partial x_1} \\
&+ \left[\overline{u_1'u_2'u_1'^2} - 3\,\overline{u_1'u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\rho}\frac{\partial \overline{\rho}}{\partial x_2} \\
&+ \left[\overline{u_1'u_3'u_1'^2} - 3\,\overline{u_1'u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\rho}\frac{\partial \overline{\rho}}{\partial x_3} \\
&= 0
\end{aligned}
$$

~~Directional Turbulent Energy Fluxes~~ and

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'^2}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'^2}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'^2}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'^2}\right]$$

$$+\ 2\left[\overline{u_1'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_3}\right]$$

$$+\ \overline{u_2'u_2'^2}\frac{\partial\overline{u_1}}{\partial x_2}+\overline{u_1'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_3}$$

$$-\ 2\,\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$

$$-\ \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+\ 2\left[\overline{\overline{c_2^2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+\ \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]$$

$$+\ 2\left[\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+\ \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{c_2^2}\right]+\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{c_2^2}\right]+\overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{c_1^2}\right]$$

$$+\ \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_2'^2}\right]$$

$$+\ \left[\overline{u_1'u_2'u_2'^2}-3\,\overline{u_1'u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_2}$$

$$+\ \left[\overline{u_1'^2u_2'^2}-\overline{u_1'^2}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_2'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_1}$$

$$+\ \left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_3}$$

$$=0$$

Structure Correlations

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u}_1 \frac{\partial}{\partial x_1}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u}_2 \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_3'}\right] + \overline{u}_3 \frac{\partial}{\partial x_3}\left[\overline{u_1' u_2' u_3'}\right] \\
&+ \overline{u_1' u_2' u_3'} \frac{\partial \overline{u}_1}{\partial x_1} + \overline{u_3' u_2'^2} \frac{\partial \overline{u}_2}{\partial x_2} + \overline{u_2' u_3'^2} \frac{\partial \overline{u}_1}{\partial x_3} \\
&+ \overline{u_3' u_1'^2} \frac{\partial \overline{u}_2}{\partial x_1} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u}_2}{\partial x_2} + \overline{u_1' u_3'^2} \frac{\partial \overline{u}_2}{\partial x_3} \\
&+ \overline{u_2' u_1'^2} \frac{\partial \overline{u}_3}{\partial x_1} + \overline{u_1' u_2'^2} \frac{\partial \overline{u}_2}{\partial x_3} + \overline{u_1' u_2' u_3'} \frac{\partial \overline{u}_3}{\partial x_3} \\
&- \overline{u_1' u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1' u_3'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2' u_3'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right] \\
&- \overline{u_1' u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right]\right] \\
&- \overline{u_2' u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right] \\
&+ \overline{u_1'^2} \frac{\partial}{\partial x_1}\left[\overline{\overline{c_2 c_3}}\right] + \overline{u_1' u_2'} \frac{\partial}{\partial x_2}\left[\overline{\overline{c_2 c_3}}\right] + \overline{u_1' u_3'} \frac{\partial}{\partial x_3}\left[\overline{\overline{c_2 c_3}}\right] \\
&+ \overline{u_1' u_2'} \frac{\partial}{\partial x_1}\left[\overline{\overline{c_1 c_3}}\right] + \overline{u_2'^2} \frac{\partial}{\partial x_2}\left[\overline{\overline{c_1 c_3}}\right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_3}\left[\overline{\overline{c_1 c_3}}\right] \\
&+ \overline{u_1' u_3'} \frac{\partial}{\partial x_1}\left[\overline{\overline{c_1 c_2}}\right] + \overline{u_2' u_3'} \frac{\partial}{\partial x_2}\left[\overline{\overline{c_1 c_2}}\right] + \overline{u_3'^2} d \frac{\partial}{\partial x_3}\left[\overline{\overline{c_1 c_2}}\right] \\
&+ \overline{\overline{c_1^2}} \frac{\partial}{\partial x_1}\left[\overline{u_2' u_3'}\right] + \overline{\overline{c_1 c_2}} \frac{\partial}{\partial x_2}\left[\overline{u_2' u_3'}\right] + \overline{\overline{c_1 c_3}} \frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right] \\
&+ \overline{\overline{c_1 c_3}} \frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_2 c_3}} \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_3^2}} \frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right] \\
&+ \overline{\overline{c_1 c_2}} \frac{\partial}{\partial x_1}\left[\overline{u_1' u_3'}\right] + \overline{\overline{c_2^2}} \frac{\partial}{\partial x_2}\left[\overline{u_1' u_3'}\right] + \overline{\overline{c_2 c_3}} \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right] \\
&+ \frac{\partial}{\partial x_1}\left[\overline{u_2' u_3' u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_3' u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_2' u_3'^2}\right] \\
&+ \left[\overline{u_2' u_3' u_1'^2} - \overline{u_2' u_3'}\left[\overline{u_1'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_1' u_3'}\right]\right] \frac{1}{\rho} \frac{\partial \overline{\rho}}{\partial x_1} \\
&+ \left[\overline{u_1' u_3' u_2'^2} - \overline{u_1' u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_2' u_3'}\right]\right] \frac{1}{\rho} \frac{\partial \overline{\rho}}{\partial x_2} \\
&+ \left[\overline{u_1' u_2' u_3'^2} - \overline{u_1' u_2'}\left[\overline{u_3'^2}\right] - 2\,\overline{u_1' u_3'}\left[\overline{u_2' u_3'}\right]\right] \frac{1}{\rho} \frac{\partial \overline{\rho}}{\partial x_3} \\
&= 0
\end{aligned}
$$

5-2. Replace claim 20 as follows:

20. (Currently Amended) A computer readable storage medium containing a set of instructions for a general purpose computer, the set of instructions defining a method of deriving a set of closed time average turbulent moment equations for ~~modelling~~ analyzing and computing anisotropic turbulent flow quantities ~~in~~ of an anisotropic fluid comprising:

defining, for an anisotropic fluid, a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

calculating n^{th} order , wherein n is odd, directional kinetic energy fluxes and structure correlation equations ~~closure relationships~~ using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

using the set of closed time average turbulent moment equations to ~~predict~~ calculate a ~~anisotropic~~ turbulent flow quantity of the anisotropic fluid; and

displaying the calculated turbulent flow quantity;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional ~~Turbulent~~ Kinetic Energy

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]\\
&+\ 2\left[\overline{u_1'^2}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_1'u_2'}\frac{\partial\overline{u_1}}{\partial x_2}+\overline{u_1'u_3'}\frac{\partial\overline{u_1}}{\partial x_3}\right]\\
&+\ \frac{1}{\rho}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{\rho u_2'u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{\rho u_3'u_1'^2}\right]\right]\\
&=\ 0
\end{aligned}
$$

~~Turbulent~~ Shear

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\\
&+\ \overline{u_1'u_2'}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_2'^2}\frac{\partial\overline{u_1}}{\partial x_2}+\overline{u_2'u_3'}\frac{\partial\overline{u_1}}{\partial x_3}\\
&+\ \overline{u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_1'u_3'}\frac{\partial\overline{u_2}}{\partial x_3}\\
&+\ \frac{1}{\rho}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{\rho u_1'u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{\rho u_1'u_2'u_3'}\right]\right]\\
&=\ 0
\end{aligned}
$$

Directional ~~Turbulent~~ Kinetic Energy Fluxes

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1' u_1'^2}\right] + \overline{u}_1 \frac{\partial}{\partial x_1}\left[\overline{u_1' u_1'^2}\right] + \overline{u}_2 \frac{\partial}{\partial x_2}\left[\overline{u_1' u_1'^2}\right] + \overline{u}_3 \frac{\partial}{\partial x_3}\left[\overline{u_1' u_1'^2}\right] \\
&+ 3\left[\overline{u_1' u_1'^2}\,\frac{\partial \overline{u}_1}{\partial x_1} + \overline{u_2' u_1'^2}\,\frac{\partial \overline{u}_1}{\partial x_2} + \overline{u_3' u_1'^2}\,\frac{\partial \overline{u}_1}{\partial x_3}\right] \\
&- 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right] \\
&+ 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right] \\
&+ 3\left[\overline{u_1'^2}\,\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1' u_2'}\,\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1' u_3'}\,\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right] \\
&+ \frac{\partial}{\partial x_1}\left[\overline{u_1'^2 u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3' u_1'^2}\right] \\
&+ \left[\overline{u_1'^2 u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1} \\
&+ \left[\overline{u_1' u_2' u_1'^2} - 3\,\overline{u_1' u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2} \\
&+ \left[\overline{u_1' u_3' u_1'^2} - 3\,\overline{u_1' u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3} \\
&= 0
\end{aligned}
$$

~~Directional Turbulent Energy Fluxes~~and

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'u_2'^2}\right]+\overline{u}_2\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'^2}\right]+\overline{u}_1\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'^2}\right]+\overline{u}_3\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'^2}\right]\\
&+\ 2\left[\overline{u_1'u_2'^2}\,\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_1'^2}\,\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\,\frac{\partial\overline{u_2}}{\partial x_3}\right]\\
&+\ \overline{u_2'u_2'^2}\,\frac{\partial\overline{u_1}}{\partial x_2}+\overline{u_1'u_1'^2}\,\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_1'^2}\,\frac{\partial\overline{u_1}}{\partial x_3}\\
&-\ 2\,\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]\\
&-\ \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]\\
&+\ 2\left[\overline{\overline{c_2^2}}\,\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_1c_2}}\,\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\,\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]\\
&+\ \overline{\overline{c_1c_2}}\,\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1^2}}\,\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1c_3}}\,\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]\\
&+\ 2\left[\overline{u_2'^2}\,\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{u_1'u_2'}\,\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{u_2'u_3'}\,\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]\\
&+\ \overline{u_1'u_2'}\,\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2^2}}\right]+\overline{u_1'^2}\,\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2^2}}\right]+\overline{u_1'u_3'}\,\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\\
&+\ \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_2'^2}\right]\\
&+\ \left[\overline{u_1'u_2'u_2'^2}-3\,\overline{u_1'u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\rho}}\,\frac{\partial\overline{\rho}}{\partial x_2}\\
&+\ \left[\overline{u_1'^2u_2'^2}-\overline{u_1'^2}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_2'}\right]\right]\frac{1}{\overline{\rho}}\,\frac{\partial\overline{\rho}}{\partial x_1}\\
&+\ \left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\,\frac{\partial\overline{\rho}}{\partial x_3}\\
&=0
\end{aligned}
$$

Structure Correlations

$$
\begin{aligned}
&\frac{\partial}{\partial t}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'}\right]\\
&+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_3'^2}\frac{\partial\overline{u_1}}{\partial x_3}\\
&+\overline{u_3'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_1'u_3'^2}\frac{\partial\overline{u_2}}{\partial x_3}\\
&+\overline{u_2'u_1'^2}\frac{\partial\overline{u_3}}{\partial x_1}+\overline{u_1'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_3}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_3}}{\partial x_3}\\
&-\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right]\\
&-\overline{u_1'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]\\
&-\overline{u_2'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]\\
&+\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_2c_3}}\right]\\
&+\overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_3}}\right]\\
&+\overline{u_1'u_3'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_3'^2}d\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_2}}\right]\\
&+\overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\\
&+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_3^2}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\\
&+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2^2}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\\
&+\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'^2}\right]\\
&+\left[\overline{u_2'u_3'u_1'^2}-\overline{u_2'u_3'}\left[\overline{u_1'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_1}\\
&+\left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_2}\\
&+\left[\overline{u_1'u_2'u_3'^2}-\overline{u_1'u_2'}\left[\overline{u_3'^2}\right]-2\,\overline{u_1'u_3'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_3}\\
&=0
\end{aligned}
$$

Reasons for Allowance

6. The following is an Examiner's statement of reasons for allowance:

6-1. The closest prior art of record discloses:

(1) Deriving a solution of the Boltzmann equation for a gas with unequal directional kinetic energies (Kliegel, "Maxwell Boltzmann Gas Dynamics").

(2) A simple anisotropic turbulent flow model (Rylov et al., "A Simple Model for Anisotropic Turbulent Flow in Open Channels").

6-2. Independent claim 19 is directed at a method of analyzing and computing anisotropic turbulent flow quantities of an anisotropic fluid. This independent claim identifies the distinct combination of features including "calculate n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations", "using the set of closed time average turbulent moment equations to calculate a turbulent flow quantity of the anisotropic fluid", and a set of moment equations, which has not been uncovered in a single teaching, nor would a modification of prior art references be obvious to one of ordinary skill in the art to yield these limitations in the context of the claim. Claim 19 is deemed allowable.

6-3. Independent claim 20 is a computer-readable storage medium claim reciting equivalent method limitations as in the allowable claim 19 and is deemed allowable for the same reason as claim 19.

7. Any comments considered necessary by applicant must be submitted no later than the payment of the issue fee and, to avoid processing delays, should preferably accompany the issue

fee. Such submissions should be clearly labeled "Comments on Statement of Reasons for Allowance."

Conclusion

8. Any inquiry concerning this communication or earlier communications from the Examiner should be directed to Herng-der Day whose telephone number is (571) 272-3777. The Examiner can normally be reached on 9:00 - 17:30.

Any inquiry of a general nature or relating to the status of this application should be directed to the TC 2100 Group receptionist: (571) 272-2100.

If attempts to reach the Examiner by telephone are unsuccessful, the Examiner's supervisor, Kamini S. Shah can be reached on (571) 272-2279. The fax phone numbers for the organization where this application or proceeding is assigned is (571) 273-8300.

Information regarding the status of an application may be obtained from the Patent Application Information Retrieval (PAIR) system. Status information for published applications may be obtained from either Private PAIR or Public PAIR. Status information for unpublished applications is available through Private PAIR only. For more information about the PAIR system, see http://pair-direct.uspto.gov. Should you have questions on access to the Private PAIR system, contact the Electronic Business Center (EBC) at 866-217-9197 (toll-free).

Herng-der Day H.D.
June 22, 2006

KAMINI SHAH
SUPERVISORY PATENT EXAMINER